Dept. of Math. University of Oslo
Statistical Research Report No. 4
ISSN 0806–3842 June 2007

Bankruptcy prediction in Norway: a comparison study

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July 26, 2007

Abstract

In this paper we develop statistical models for bankruptcy prediction of Norwegian firms in the limited liability sector using annual balance sheet information. We fit generalized linear-, generalized linear mixed- and generalized additive models in a discrete hazard setting. It is demonstrated that careful examination of the functional relationship between the explanatory variables and the probability of bankruptcy enhances the models' forecasting performance. Using information on the industry sector we model the unobserved heterogeneity between different sectors through an industry-specific random factor in the generalized linear mixed model. The models developed in this paper are shown to outperform the model with Altman's variables at all levels of risk. As a measure of models' forecasting accuracy the area under the ROC curve is used.

KEYWORDS: Bankruptcy Prediction, Industry Eects, Hazard Model, Generalized Linear Model, Generalized Linear Mixed Model, Generalized Additive Model

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1 Introduction

Bankruptcy prediction is attracting the attention of both academics and practitioners since the seminal works of Beaver (1966) and Altman (1968) in the late 1960s. Researchers traditionally rely on linear combinations of financial ratios as predictors and use a single observation per company. Several recent studies emphasize that the relationship between explanatory variables and the logit (probit) of the default probability is often *non-linear* (see Sobehart and Stein (2000), Falkenstein et al. (2000), Berg (2007)). Additionally, Shumway (2001), Chava and Jarrow (2004) and Hillegeist et al. (2004) emphasize that a single-period approach neglects important information when a company is at risk but remains solvent. To avoid these deficiencies of the traditional approaches, suggestions have been made to use neural networks or generalized additive models to model non-linearities and hazard models instead of single-period static models to incorporate information from the complete period at risk (see Shumway (2001), Chava and Jarrow (2004) and Hillegeist et al. (2004)). In this paper we follow the pattern of non-linear modeling and evaluate the forecast performance of both static- and hazard models. Our models are broad in scope in the sense that they apply to all industry sectors, including financial institutions. Additionally, we model the unobserved heterogeneity between different industry sectors by introducing an industry-specific intercept as a random factor in our non-linear logistic regression.

The purpose of this paper is to develop statistical models for bankruptcy prediction of firms in the limited liability sector of Norway. The $98,421$ firms in our database are observed on an annual basis and most of them are not registered on any exchange. Therefore, we have to rely on traditional accounting-based methods. We examine whether one can enhance bankruptcy prediction accuracy by a careful examination of the functional relationship between explanatory variables and the probability of bankruptcy. We utilize generalized additive models (GAM) in exploratory analysis to reveal non-linear relations to be used in the generalized linear model (GLM). Further, we show that when one carefully models, through linear and non-linear transformations of covariates in GLM, prediction accuracies of GLM and GAM are approximately the same. A slight improvement of model performance is further obtained by estimating an industry-specific intercept as a random factor. In the assessment of model accuracy we use ROC and CAP curve analysis, which became widely accepted in the bankruptcy prediction literature since they were introduced in Sobehart et al. (2000) and Sobehart and Keenan (2001). Forecasting ability of our models is stable over different subsets of the dataset and over time. The models are then compared to the celebrated Altman's Z-score model which uses a linear combination of 5 financial ratios as a proxy of the default probability. The Altman's model is reestimated and shown to capture less publicly-available information than the models we use in our analysis for this specific dataset. Improvements obtained by using a hazard instead of static setting are minimal, possibly because the maximal period at risk in our sample is low compared to previous studies of hazard models.

The paper is organized as follows. In Section 2 we give a brief historical overview of bankruptcy prediction methods and outline statistical methods used in our analysis. Section 3 describes our bankruptcy database. In Section 4 we present the fitted models and evaluate their out-of-time prediction performance.

Section 5 concludes and discusses open problems for further research.

2 Statistical models for bankruptcy prediction

The bankruptcy prediction literature involves a number of statistical techniques used to obtain reliable estimates of default probability. The studies of Beaver (1966) and Altman (1968) that employ univariate and multivariate discriminant analysis respectively, are considered pioneering investigations of the relationship between the financial status of a company and its probability of failure. Subsequently, new statistical methods including gambler's ruin and option pricing theory, as well as linear regression, have been successfully applied in empirical analysis (see Wilcox (1971), Merton (1974), Martin (1977), Ohlson (1980), Zmijewski (1984)). Techniques used nowadays to construct bankruptcy prediction models involve neural and Bayesian networks (Tam and Kiang (1992), Sun and Shenoy (2007)), theory of point processes (Das et al. (2007), Duffie et al. (2007)), support vector machines (Härdle et al. (2005)), and many others. For extensive reviews of related literature the reader is referred to Altman and Hotchkiss (2005), Altman and Narayanan (1997) or Falkenstein et al. (2000). We focus our attention on methods that emphasize the use of survival analysis and industry effects in failure prediction.

Recently, several studies including Shumway (2001), Chava and Jarrow (2004), Hillegeist et al. (2004) have indicated that conventional models have a drawback of being based on the utilization of only a single observation per company. Traditionally the default probability of a company, irrespective of its bankruptcy status, has been dependent solely on its last available set of predictors. Such models, often called static, are shown to be outperformed by dynamical hazard models that incorporate the financial history of a company from the entire observation period. Applications of survival analysis techniques in bank failure prediction has a long history (see LeClere (2000) and Haling and Hayden (2006) for a review), while in bankruptcy prediction of non-financial institutions these methods have been disregarded since the work of Shumway (2001). Shumway argues that information neglected by static models can significantly improve model's forecasting accuracy, and highlights the simplicity of maximum likelihood estimation in the dynamical framework. Since the discrete hazard model plays one of the central roles in our investigation, we briefly outline its setting.

Assume that each firm i in the study has a failure time T_i and a censoring time C_i, both observed at discrete times, and that T_i, C_i are independent random variables with values in $\{1, \ldots, k\}$, where k denotes the end of the observation period. The observable lifetime of a firm i is then $S_i = \min(T_i, C_i)$. Let Δ_i denote the random censoring indicator given by

$$\Delta_i = \begin{cases} 1, & T_i \le C_i, \qquad \text{(non-censored)}, \\ 0, & T_i > C_i, \qquad \text{(censored)}. \end{cases}$$

In addition to the observed lifetime s_i, we consider firm-specific time-varying covariates $\mathbf{x}_{it} \in \mathbb{R}^p$, that are assumed to have an influence on the lifetime. The data is given by

$$(s_i, \delta_i, \mathbf{x}_i(s_i)), \qquad i = 1, \ldots, n,$$

where $\mathbf{x}_i^{'}(s_i) := (\mathbf{x}_{i1}^{'}, \ldots, \mathbf{x}_{is_i}^{'})$ is the history of firm i until the observed lifetime s_i, and δ_i is the observed censoring indicator.

The basic quantity characterizing S_i is the *discrete hazard function*

$$\lambda(t|\,\mathbf{x}_i(t)) := \mathrm{P}(S_i = t|\, S_i > t-1, \mathbf{x}_i(t)), \qquad t \in \{1, \ldots, k\}, \tag{1}$$

which is assumed to be dependent on parameters or functions to be estimated. The exact form of the dependence of the hazard rate $\lambda(t|\,\mathbf{x}_i(t))$ on time-varying firm-specific covariates is given in Section 4.2. Under certain conditions, parameters of dynamical hazard model can be estimated in the framework of ordinary binary regression by treating the annual bankruptcy indicators as independent binomials (see Fahrmeir and Tutz (2001) or Shumway (2001)). Precise assumptions under which the correspondence between the two models holds can be found in Arjas and Haara (1987) or Fahrmeir and Tutz (2001, p. 396). We note here that the hazard model built on only one year of data coincides with the static model.

We conclude this section with a short outline of research where the significance of industry effects in bankruptcy prediction modeling was discussed. Plat and Plat (1990, 1991) are among the first studies that illustrate the importance of industry-relative adjustments in failure prediction. Subsequently, a number of papers documented the impact of industry groupings on bankruptcy announcements. Lang and Stulz (1992) examine contagion and competitive intra-industry effects on default rate, while Alfo et al. (2005) use random industry effects to anticipate problematic firms. For a detailed review of reasons for presence of industry-specific information in bankruptcy prediction models, and an extensive list of references where these reasons are elaborated, the reader is referred to Chava and Jarrow (2004).

3 Data set

3.1 The Data Set

Financial statements and bankruptcy status for limited-liability firms in Norway are observed on an annual basis in the time period 1996-2000. Firms reporting non-positive total assets were eliminated. Balance sheets with book equity, short term debt or revenue from operations equal to 0 were excluded from further investigation in order to avoid null divisions when calculating financial ratios. Exploratory data analysis indicated a substantial lag between the date of the last reported financial statement and the bankruptcy date. This phenomenon is also described in Bernhardsen (2001). Among companies that were declared bankrupt in the time-period 1997-2001, only 25% report their financial statements in the last year of their existence, while for the remaining 75% we observe at least one year of missing data. For this reason, all companies (bankrupt as well as non-bankrupt) with missing financial statements for at least one year before bankruptcy or the end of the observation period were excluded from further analysis. Since salaried household work and internal organs and organizations were represented by only 3, respectively 1 firm in the resulting sample, these two industry sectors were not considered in our paper. For each of the continuous covariates used to estimate the model, the values below 0.2%-quantile and above 99.8%-quantile were calculated, and firms with these financial statements were also excluded from further consideration. Truncation of the data is often performed in order to remove outliers that frequently occur due to typos or recording errors (see Shumway (2001), Chava and Jarrow (2004)). Note that the discrete hazard model can be estimated in the framework of binary regression, since censoring can occur only at the end of the observation period due to the data requirements. Our final sample consists of $436,145$ firm-years corresponding to $98,421$ unique firms, and contains $2,270$ bankruptcies.

3.2 Explanatory variables

The set of covariates included in our model building process combines conventional accounting ratios used in bankruptcy prediction studies, and covariates traditionally employed in the credit risk analysis at Norges Bank, presented in Bernhardsen (2001). We take into account 5 frequently used default risk factors: profitability, solidity, liquidity, size and leverage. Additionally, we include industry indicator variables, information on the number of auditor remarks, age of a company, and an indicator of dividends paid current year as predictors of default probability. The list of time-varying explanatory variables considered in our analysis consists of

(1) REVANM_{it} – the number of auditor remarks of firm i at time t,

(2) AGE_{it} – age of a firm i at time t measured in years,

(3) DIV_{it} – indicator for dividends paid by firm i at time t (dichotomous),

(4) EKA_{it} – book value of equity to total assets of firm i at time t (solidity),

(5) SIZE_{it} – logarithm of total assets of firm i at time t (size),

(6) CashR_{it} – cash and marketable securities to current liabilities of firm i at time t (liquidity),

(7) RetAss_{it} – return on assets to total assets of firm i at time t (profitability),

(8) CLTA_{it} – current liabilities to total assets of firm i at time t (leverage).

Here $i = 1, \ldots, n, t = t_0(i), \ldots, S_0(i)$, where $t_0(i)$ and $S_0(i)$ denote the starting and survival time of firm i. In addition, the information about the sector a firm belongs to (fixed over time) is included into the model through an industry-specific intercept estimated as a fixed or random factor. The distribution of firms and bankruptcies with respect to the industry sector is given in Table 1.

In the remainder of this section we shortly discuss properties of covariates described above placing the emphasis on the difference between bankrupt and solvent firms.

Summary statistics for REVANM can be found in Table 2. The table indicates that distributions of the number of auditor remarks among solvent and bankrupt firms are distinct. Additionally, we notice the change in the distribution in 1999, when the percentage of companies with more then one remark becomes lower compared to the period 1996–1998. This change is possibly due to the fact that prior to 1998 Norwegian law was imposing only moderate sanctions for non-reporting financial information while more stringent regulations were introduced in 1998.

Industry sector	Total	Bankrupt	Bankrupt(%)
Forestry and agriculture	607	17	2.80
Fishing	1,059	18	1.70
Mining and extraction	599	6	1.00
Industry	9,810	315	3.21
Water and power supply	300	1	0.33
Building and construction	8,503	242	2.85
Commodity trade, vehicle and domestic appliance repair	26,340	863	3.28
Hotel and catering activity	3,356	224	6.67
Transport and communication	5,616	123	2.19
Finance and insurance	4,225	22	0.52
Property operations, rental business and commercial services	32,373	348	1.07
Public administration	46	2	4.35
Education	558	12	2.15
Health and social service	2,086	20	0.96
Other social and personal services	2,943	57	1.94
Total	98,421	2,270	2.31

Table 1: Distribution of firms with respect to the industry sector.

In Figure 1 the histogram of AGE (bellow 50 years) and kernel density estimators of covariates EKA, SIZE, RetAss, CashR and CLTA [1] for bankrupt and non-bankrupt firms in the complete data-set are given. The difference between bankrupt and solvent companies is clearly visible. For all covariates except of CashR, the difference in modes of the respective distributions is evident. In agreement with econometric intuition, we observe that bankrupt companies are more likely to have low values of EKA, RetAss, CashR and high values of CLTA. Notice that for all covariates except of SIZE, the shape of the density of bankrupt firms differs from respective shape estimated within the group of solvent companies. Further empirical analysis (not presented here) shows that distributions of all covariates except of REVANM are stable over time, and that indicator of dividends payed can be seen as a potentially powerful predictor of failure.

4 Results

The aim of this section is to describe the models fitted to our dataset and evaluate and compare their forecasting performance. We fit a GLM, transforming covariates according to the exploratory analysis in Section 4.1. We compare it to the generalized linear mixed model (GLMM) with random, industry-specific intercept, and a GAM. All models are estimated using the same set of explanatory variables, presented in Section 3.2. The three models (GLM, GLMM,

[1]Since continuous covariates that we consider have heavy-tailed distributions, the kernel density estimators are given for values of EKA above 2%-quantile, RetAss between 1%- and 99%-quantile, CashR and CLTA below 80%- and 98%-quantile , respectively.

Year	Status	0	1	2	3	≥ 4
1996	NB	75.42	22.38	0.57	1.46	1.63
	B	11.85	64.22	0.71	18.25	23.22
1997	NB	76.33	21.63	0.52	1.38	1.52
	B	15.33	59.80	1.26	18.59	23.62
1998	NB	76.03	21.93	0.55	1.35	1.49
	B	13.82	63.09	1.09	17.09	22.00
1999	NB	81.58	17.39	0.85	0.02	0.18
	B	23.17	67.20	6.60	0.18	3.03
2000	NB	81.82	17.22	0.81	0.00	0.15
	B	23.89	68.14	5.90	0.00	2.06

Table 2: Percentage of companies with respect to the number of auditors' remarks for non-bankrupt (NB) and bankrupt (B) firms separately.

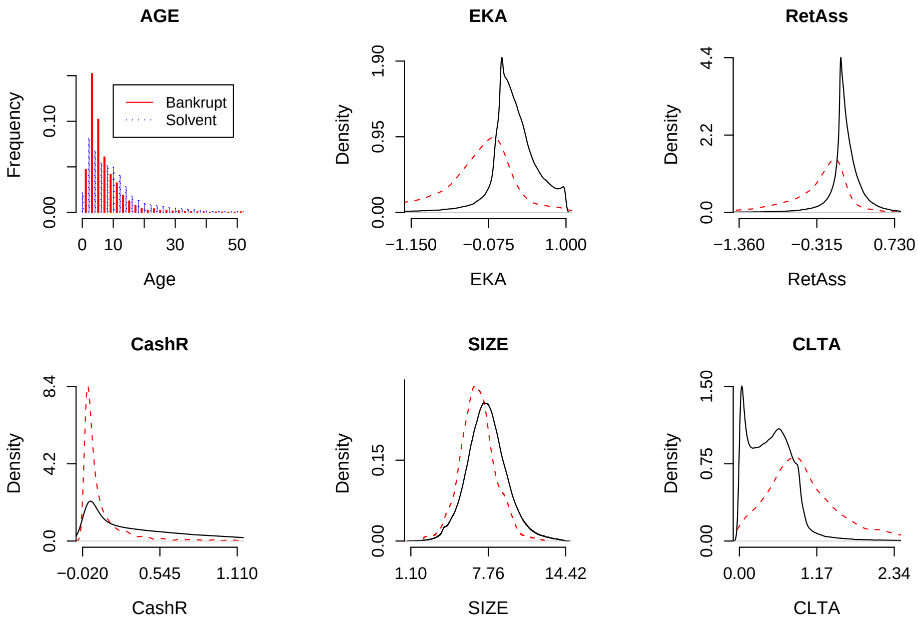

Figure 1: Histogram of AGE. Kernel density estimators of EKA, RetAss, CashR, SIZE and CLTA for bankrupt (dashed line) and solvent (solid line) firms separately.

GAM) are estimated and validated on different subsets of the complete sample. To illustrate the discriminative power of our models, we fit them using financial statements 1996-1999, predict default probabilities in 2000, and construct kernel density estimates of their logits for bankrupt and solvent firms, respectively. Finally, we estimate a GLM with Altman's variables (see Altman (1968)), using only linear transformations of covariates, and compare it to the models described above. All four models are estimated in a static and hazard setting, respectively. Their forecasting accuracy does not improve substantially if the hazard model is used instead of the static one.

4.1 Exploratory Data Analysis

In order to make inference about the form of the functional relationship between the logit of the hazard rate (1) and continuous explanatory variables, we fit the GAM

$$
\begin{aligned}
\operatorname{logit} \lambda(t|\,\mathbf{x}_{it}) = \beta_1 \mathrm{REVANM}_{it} + \beta_2 \mathrm{AGE}_{it} + \sum_{j=1}^{15} \beta_{3j} D_{ji} + \beta_4 \mathrm{DIV}_{it} \\
+ s_5(\mathrm{CashR}_{it}) + s_6(\mathrm{CLTA}_{it}) \\
+ s_7(\mathrm{SIZE}_{it}) + s_8(\mathrm{RetAss}_{it}) + s_9(\mathrm{EKA}_{it})
\end{aligned}
\tag{2}
$$

to the complete dataset. Here $D_{ji},\ j = 1, \ldots, 15$, are dummy variables being 1 if firm i belongs to industry j and 0 otherwise and $\hat{s}_5, \ldots, \hat{s}_9$ are smoothing splines to be estimated. We observe non-linear relations in the spline terms $\hat{s}_5, \hat{s}_7, \hat{s}_8$ and $\hat{s}_9$, i.e. for variables CashR, SIZE, RetAss and EKA. The $\hat{s}_6$ term, describing the effect of CLTA on default probability, can be considered as linear. The forms of the estimated functions are depicted in Figures 2 and 3. For more details regarding the theory of GAM, the reader is referred to Hastie and Tibshirani (1990).

In Figure 2 functions $\hat{s}_5, \hat{s}_6$ and $\hat{s}_7$ are plotted. Function $\hat{s}_5$ is plotted for values of CashR lower than the 90%-quantile, since above that value the form of $\hat{s}_5$ becomes unstable, possibly due to outliers. The plot suggests to use the function $\exp(-\mathrm{CashR})$ in a corresponding GLM model. Similarly, $\hat{s}_6$ is depicted for values of CLTA below the 98%-quantile. In that range, the estimated function can be considered as linear, and therefore CLTA enters linearly in our final GLM model. Function $\hat{s}_7$ is plotted on the whole range of the covariate SIZE, and we decide to use a polynomial of degree 2 to model its influence on the default probability.

Visual examination of the left plots in Figure 3, where $\hat{s}_8$ and $\hat{s}_9$ are plotted for the entire range of variables RetAss and EKA respectively, suggests that possibly separate functions should be fitted for negative and positive values for two of the covariates considered. In the middle and right hand side plots of Figure 3, the estimated functions are depicted on the negative and positive half axes, respectively. We decide to use a polynomial of degree 3(2) for modeling the influence of negative (positive) values of RetAss, while the effect of EKA is modeled by 2 separate polynomials of degree 2, each fitted on the corresponding half axis.

We emphasize here that both the shape of functions we use in our analysis and the list of explanatory variables should be seen as suggestion. Our recommendation is to carefully investigate functional relationship of covariates to

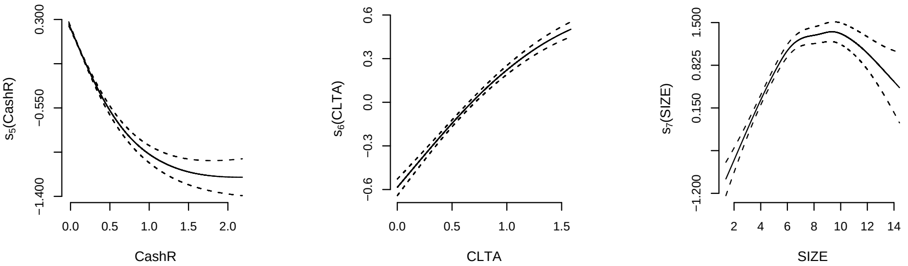

Figure 2: Estimated transformations of CashR (left, values of CashR below 90%-quantile), CLTA (middle, values of CLTA below 98%-quantile), and SIZE (right).

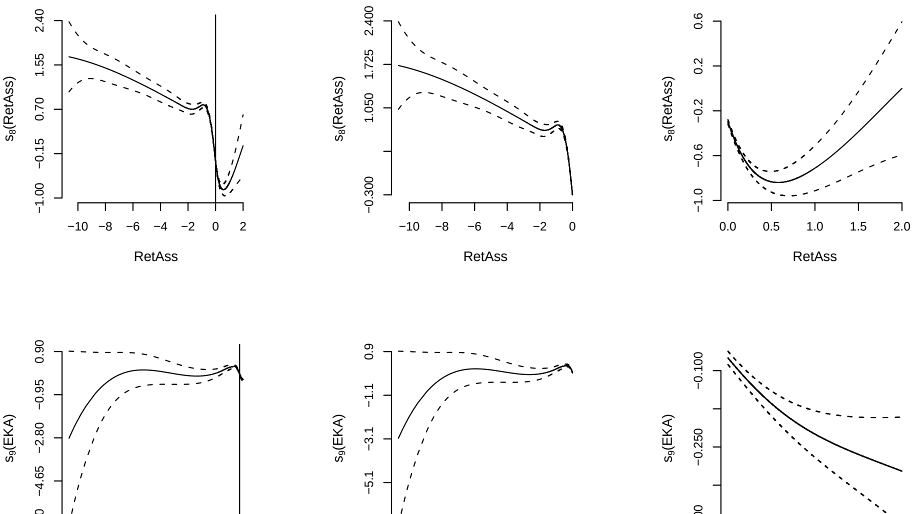

Figure 3: Estimated transformation of RetAss (upper) and EKA (lower). Whole range of RetAss, EKA (left), negative values (middle), positive values (right).

default probability, and use the results of exploratory data analysis to build the final model.

4.2 Fitted models

We assume that the discrete hazard rate (1) depends only on the last value of the covariates [2], namely

$$\lambda(t|\mathbf{x}_i(t)) := \lambda(t|\mathbf{x}_{it}),$$

and consider the following three models for $\lambda(t|\mathbf{x}_{it})$.

Model 1: GLM

The GLM has the form indicated in Section 4.1, namely

$$\begin{aligned}
\mathrm{logit}\lambda(r|\,\mathbf{x}_{it}) = {} & \beta_1 I(\mathrm{REVANM}_{it} > 0) + \beta_2 \mathrm{AGE}_{it} + \sum_{j=1}^{15} \beta_{3j} D_{ji} + \beta_4 \mathrm{DIV}_{it} \\
& + \beta_{51} I(\mathrm{EKA}_{it} \geq 0)\mathrm{EKA}_{it} + \beta_{52} I(\mathrm{EKA}_{it} \geq 0)\mathrm{EKA}_{it}^2 \\
& + \beta_{53} I(\mathrm{EKA}_{it} < 0)\mathrm{EKA}_{it} + \beta_{54} I(\mathrm{EKA}_{it} < 0)\mathrm{EKA}_{it}^2 \\
& + \beta_{61}\mathrm{SIZE}_{it} + \beta_{62}\mathrm{SIZE}_{it}^2 + \beta_{71} I(\exp(-\mathrm{CashR}_{it})) \\
& + \beta_{81} I(\mathrm{RetAss}_{it} \geq 0)\mathrm{RetAss}_{it} + \beta_{82} I(\mathrm{RetAss}_{it} \geq 0)\mathrm{RetAss}_{it}^2 \\
& + \beta_{83} I(\mathrm{RetAss}_{it} < 0)\mathrm{RetAss}_{it} + \beta_{84} I(\mathrm{RetAss}_{it} < 0)\mathrm{RetAss}_{it}^2 \\
& + \beta_{85} I(\mathrm{RetAss}_{it} < 0)\mathrm{RetAss}_{it}^3 + \beta_9 \mathrm{CLTA}_{it}
\end{aligned} \tag{3}$$

We remark here that all coefficients included in Model (3) were significant at the 5%-level when being estimated from the complete dataset. Analysis not reported here show that interactions of continuous ratios with industry sector indicators are not significant. Therefore industry-specific slopes for EKA, RetAss, CashR, SIZE and CLTA are not included into our model.

Model 2: GLMM

In order to incorporate homogeneity within industry sectors, while allowing for heterogeneity between different sectors, we fit the GLMM with random industry-specific intercept

$$\begin{aligned}
\mathrm{logit}\lambda(t|\,\mathbf{x}_{it}) = {} & \beta_1 I(\mathrm{REVANM}_{it} > 0) + \beta_2 \mathrm{AGE}_{it} + \sum_{j=1}^{15} b_j D_{ji} + \beta_4 \mathrm{DIV}_{it} \\
& + \beta_{51} I(\mathrm{EKA}_{it} \geq 0)\mathrm{EKA}_{it} + \beta_{52} I(\mathrm{EKA}_{it} \geq 0)\mathrm{EKA}_{it}^2 \\
& + \beta_{53} I(\mathrm{EKA}_{it} < 0)\mathrm{EKA}_{it} + \beta_{54} I(\mathrm{EKA}_{it} < 0)\mathrm{EKA}_{it}^2 \\
& + \beta_{61}\mathrm{SIZE}_{it} + \beta_{62}\mathrm{SIZE}_{it}^2 + \beta_{71} I(\exp(-\mathrm{CashR}_{it})) \\
& + \beta_{81} I(\mathrm{RetAss}_{it} \geq 0)\mathrm{RetAss}_{it} + \beta_{82} I(\mathrm{RetAss}_{it} \geq 0)\mathrm{RetAss}_{it}^2 \\
& + \beta_{83} I(\mathrm{RetAss}_{it} < 0)\mathrm{RetAss}_{it} + \beta_{84} I(\mathrm{RetAss}_{it} < 0)\mathrm{RetAss}_{it}^2 \\
& + \beta_{85} I(\mathrm{RetAss}_{it} < 0)\mathrm{RetAss}_{it}^3 + \beta_9 \mathrm{CLTA}_{it}
\end{aligned} \tag{4}$$

[2]Other specifications of $\lambda(t|\mathbf{x}_i(t))$ that include time-lagged covariates are possible, but rarely used in practice. Results not reported here indicate that models with $\lambda(t|\mathbf{x}_i(t)) = \lambda(t|\mathbf{x}_{it}, \mathbf{x}_{it-1})$ when fitted to our dataset do not improve forecasting ability.

where $b_j \sim N(0, \sigma^2), j = 1, \ldots, 15$, are independent random variables representing the frailty effect. The model is estimated by the penalized quasi-maximum likelihood method described in Breslow and Clayton (1993).

Model 3: GAM

Additionally, we compare the previous two models to the GAM indicated in (2) which was used in the exploratory data analysis.

Model 4: Altman

Finally, our three models are compared to the static and hazard model with Altman's variables

$$\operatorname{logit} \lambda(t|\,\mathbf{x}_{it}) = \beta_0 + \beta_1 X_{1it} + \beta_2 X_{2it} + \beta_3 X_{3it} + \beta_4 X_{4it} + \beta_5 X_{5it}, \qquad (5)$$

where $X_1, \ldots, X_5$ are as defined in Altman (1968) [3].

4.3 Predictive performance

The three models, GLM, GLMM and GAM, are fitted to various subsets of our dataset, and their predictive performance is evaluated. In the assessment of model performance we use ROC and CAP curve analysis. More specifically, we use the measure of the area under the ROC curve, denoted AUC. The AUC is the area under the ROC curve and above the 45° line corresponding to the random model divided by 1/2 (the area between the ROC curve of the perfect and random model, respectively). It is a number between zero and one, one corresponding to the perfect model correctly classifying all firms and zero indicating the random model. The summary statistic of the CAP curve, the accuracy ratio AR, can be calculated directly from AUC (see Engelmann et al. (2003)).

Table 3 shows the AUC for the models GLM, GLMM and GAM, evaluated at different fitting and prediction periods. We notice that GLM and GAM perform equally well, while GLMM has a slightly better forecasting accuracy. The lowest values of AUC are obtained when bankruptcies in 1999 were predicted. This is possibly due to the change in Norwegian law regarding sanctions for non-reporting of financial statements. Apart from evaluation of forecasts one year ahead, we have also computed AUC when bankruptcy prediction is done $2, 3$ and 4 years into the future. The results are presented in the lower part of Table 3. We observe that although the forecasting accuracy of models decline when we increase the number of forthcoming years for prediction, the power of the depreciation is not very pronounced, and the performance of the models can be considered as stable even when predicting bankruptcies several years into the future.

In order to illustrate the discriminatory power of our models we estimate them using the data 1996-1999, predict default probabilities for companies at risk in 2000, and plot kernel density estimates of their logits for bankrupt and solvent firms, respectively. Results are presented in Figure 4. We note that all

[3] To construct the variables X_2 and X_4 we use the book value instead of market value of equity, since market information is unavailable.

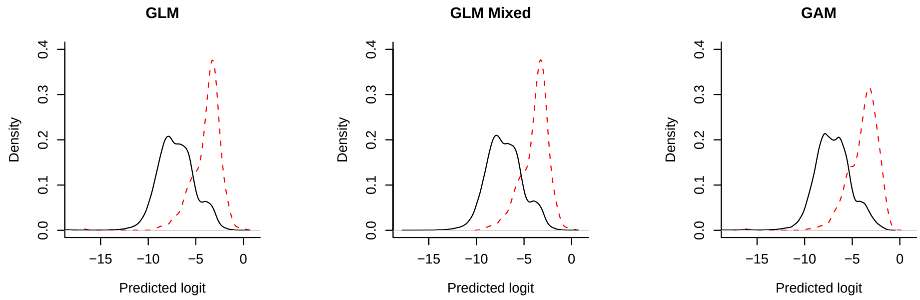

Figure 4: Kernel density estimators of logits of predicted default probabilities for data from 2000 using GLM, GLM mixed and GAM model for bankrupt (dashed line) and solvent (solid line) firms separately (1996-1999 data used to fit the model).

models have relatively high discriminatory power, and conclude that although plots obtained are suitable for illustration purposes, one needs more sensitive tools to decide which model has the highest forecasting accuracy.

We then compare the GLM, GLMM and GAM to the model with Altman's variables. Both the static and hazard models were fitted using the data from 1996-1999, and validated on the 2000 data. The corresponding ROC and CAP curves are given in Figure 5, and the AUC's are given in Table 4. We observe that the forecasting accuracy of the model with Altman's variables, in both the static- and hazard setting, is lower than the corresponding accuracy of models that include non-linear transformations of covariates. Improvements obtained by utilizing the hazard instead of the static model are not pronounced, possibly due to the fact that firms in our sample are observed only for 5 years.

5 Conclusion and discussion

This paper presents an empirical investigation of bankruptcy prediction using the GLM, GLMM and GAM, in both the static- and hazard setting. Construction of a proper default prediction model is of crucial importance to practitioners. Potential applications include credit risk analysis, development of investment guidelines and rating methodologies, among others.

We develop empirical bankruptcy prediction models for the limited liability sector in Norway over the period 1996–2000 using annual balance sheet information. Application of non-linear modeling techniques allow us to depict complex relationships between the hazard rate of a firm at risk and its time-varying covariates. The structure of the relationship is estimated using a GAM. The final GLM (3) was constructed after a careful visual inspection of the plots obtained in the exploratory data analysis. Further, the unobserved heterogeneity was taken into account by including a random industry-specific intercept into the model. We utilized the AUC to compare models. While GLM and GAM perform equally well, the GLMM is shown to have slightly higher ability to anticipate problematic firms. Comparisons of models' forecasting accuracy

Prediction 1 year ahead			
Prediction in 2000			
Data used	GLM	GLM Mixed	GAM
96, 97, 98, 99	0.899	0.901	0.900
97, 98, 99	0.899	0.901	0.901
98, 99	0.894	0.900	0.897
99	0.889	0.896	0.893
Prediction in 1999			
Data used	GLM	GLM Mixed	GAM
96, 97, 98	0.891	0.891	0.892
97, 98	0.890	0.891	0.892
98	0.891	0.891	0.893
Prediction in 1998			
Data used	GLM	GLM Mixed	GAM
96, 97	0.897	0.905	0.898
97	0.894	0.902	0.894
Prediction in 1997			
Data used	GLM	GLM Mixed	GAM
96	0.915	0.918	0.915
Prediction 2 years ahead			
Prediction in 1999 and 2000			
Data used	GLM	GLM Mixed	GAM
96, 97, 98	0.894	0.895	0.894
97, 98	0.893	0.894	0.895
98	0.892	0.894	0.894
Prediction 3 years ahead			
Prediction in 1998, 1999 and 2000			
Data used	GLM	GLM Mixed	GAM
96, 97	0.891	0.898	0.890
97	0.888	0.896	0.887
Prediction 4 years ahead			
Prediction in 1997, 1998, 1999 and 2000			
Data used	GLM	GLM Mixed	GAM
96	0.896	0.903	0.895

Table 3: Area under the ROC curve for GLM, GLM mixed and GAM model.

Model	Static	Hazard
Altman	0.816	0.830
GLM	0.897	0.899
GLM mixed	0.899	0.901
GAM	0.900	0.900

Table 4: Area under the ROC curve for model with Altman's variables, GLM, GLM mixed and GAM static and hazard model. All models are estimated using the data from 1996 until 1999, and validated on the data from 2000.

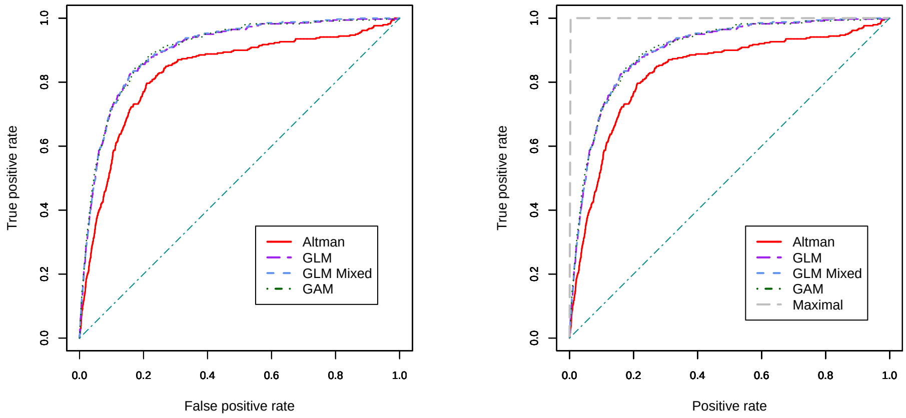

Figure 5: ROC (left) and CAP (right) curve of GLM model, GLM with random effects, GAM model and model with Altman's variables for 1-year default-horizont out-of-time prediction (1996-1999 data used to fit the model, validation on 2000 data).

were performed over different subsets of the complete sample. Utilization of the hazard instead of static setting does not improve the models' performance substantially, probably due to the fact that the maximal period at risk for firms in our dataset is only 5 years. Additionally, the model with Altman's variables was reestimated in both the hazard and traditional static setup. The AUC for the model with Altman's variables was substantially lower then the corresponding AUC of GLM, GLMM and GAM.

Future development of issues addressed in this paper may follow numerous directions. Primarily, more refined pattern in industry-effects modeling can be introduced by including information regarding the intra-industry groupings. The assumption of independence among firms may possibly be relaxed in the presence of empirical results presented in Lang and Stulz (1992), Das et al. (2007), Duffie et al. (2006) and references therein. Finally, the appropriate treatment of firms not reporting their balance sheet information, which were excluded from our analysis, should be established.

Acknowledgements

The authors thank Associate Professor Sjur Westgaard at the Department of Industrial Economy and Technology Management, Norwegian University of Science and Technology, for providing the original data set on which the entire work is based. Rada Dakovic and Claudia Czado acknowledge the support of Deutsche Forschungsgemeinschaft (CZ 86/1-1). Daniel Berg's research is supported by the Norwegian Research Council, grant number 154079/420.

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